

Abstract: Belief Selection as an Evolutionary Game with Replicator Dynamics

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The paper is a contribution to financial economics, especially to the market selection literature. It can also be viewed as a foundational contribution to the “Rebuilding of Macroeconomic Theory Project”, which is an attempt to develop a new core model for macroeconomics that is better equipped to explain or predicts periods of distress like the Great Financial Depression. I prove the following four propositions for a dynamic stochastic equilibrium model with complete markets, which is populated by Bayesian traders with heterogeneous beliefs but with a common discount factor and CRRA-utility representation: Firstly, in equilibrium, asset prices signal a “consensus belief”, defined as a wealth-weighted power mean of individual beliefs times a dispersion measure. Thus, in general, asset prices do not inform about ‘true’ relative frequencies. This also holds asymptotically, since traders start with one-period forecast distributions that are not required to ‘merge with the truth’. Secondly, under equilibrium conditions, the market selection process initiated by traders’ willingness to beat the market is completely described by the discrete-time replicator dynamics of an evolutionary game. The speed of selection is decreasing in the common coefficient of relative risk aversion. Selection pressure is decreasing in belief dispersion. Thirdly, still under equilibrium conditions, a trader’s wealth and consumption process is a non-negative strict supermartingale, if and only if the conditional fitness of its one-period forecast distribution relative to the conditional fitness of the consensus belief is strictly negative almost surely with respect to the truth (conditional fitness is defined as the negative of conditional relative entropy). Consequently, the population state is pathwise globally asymptotically stable in the replicator dynamics. Finally, given the market selection effectively conditions the consensus belief on the population state, the conditional fitness of the consensus belief converges pathwise to the asymptotically fittest belief in the belief pool. This is named the “fundamental law of market selection”, inspired by R.A. Fisher’s fundamental law of natural selection (1930) in theoretical biology, which claims that neo-Darwinian selection induces a monotonic increase over time in the average population fitness.